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ON THE SAMPLING OF PHYSICAL MATERIALS

BY

W. EDWARDS DEMING

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W. Edwards Deming
U.S. Bureau of the Budget

Scope and purpose of paper. — A very important problem in industry and commerce is the accurate, speedy, and economical measurement of the quantity and quality of physical materials, both raw and manufactured. Millions of dollars change hands every day as shipments of material are transferred from seller to buyer, or from one department to another within the same factory, or when statistics are compiled and customs duties are levied on goods imported. The aim of this paper is to mention some of the ways in which simple statistical theory may be usefully applied in the sampling of physical materials.

A distinction should be pointed out between the problems of sampling as met in (a) the statistical control of quality, in (b) acceptance-sampling, and in (c) the sampling of physical materials as encountered in this paper. In problems (a) and (b) it has always been assumed, I believe, that a sample of n manufactured articles is a so-called random sample drawn from a lot or from a half-hour's production, and that the theories of random sampling apply. Here, however, in problem (c), the question to be treated is *how to draw a sample* at random and how to draw it economically to enable some quality (weight, ash-content, average physical condition) of a lot of manufactured articles, or of manufactured or raw material, to be ascertained with a specified degree of precision, *at the lowest possible cost.* Problem (c) might be called the problem of sampling a lot of material economically.

There are several ways in which sampling may be applied for measuring some average quality of a lot of physical material.

I. The determination of the weight and quality of a shipment of material in packages which have not previously been measured may be made by weighing and testing a sample of the packages. Example: a shipment consists of hundreds of packages of wool, tobacco, sugar, or other material. What procedure of sampling will yield the weight of the shipment within ½ % and do so at minimum cost?

II. The re-determination of the average quality of a lot of material in packages which have been weighed or tested individually at some previous time. Sometimes a lot of material will have been in storage some months or years. The quality of each article or package was known at the time it was put into storage, but the quality may have changed meanwhile, or a recalibration may be necessary for any of various reasons. What is the average quality now of the entire shipment? Example: a shipment consists of hundreds of bales, sacks, or other packages of wool, tobacco, sugar, or other material,

*) Presented at the 26th Session of the International Statistical Institute, Berne, September 1949.

which were weighed one by one in some foreign port, and each weight was marked on the package in English pounds, Spanish pounds, kilograms, okes, or some other unit in which the weighing was carried out. What procedure of sampling will yield the weight or some other important quality of the shipment within ½ % and do so at minimum cost?

III. The determination of (e.g.) the average B.t.u. content or ash-content of a bulk material like a pile of coal or a carload of coal.

IV. The determination of the average physical condition of the various classes of property of a telephone system, or of a telegraph or railway system, or of a system for supplying gas or electricity to a city. This problem is important in judiciary cases involving reserve and equitable rates for service, as reserve and rates are to some extent related to the present value of the property. This problem is also important in making estimates of the type and extent of maintenance and repairs to the system that will be needed over the next 5 or 10 years. Such information enables a corporation to set aside proper amounts of money for maintenance and repairs, and, over a long-range programme, to take advantage of the market values of steel, copper, lumber, and finished materials: also to smooth out its demands for the labor that is to be required. As a sampling problem the question is, how may the samples be taken in order to evaluate the average over-all physical condition of the total property or of any subclass thereof, within (e.g.) 1 %?

General approach to a problem in sampling. — A few remarks will be made first concerning the general approach to a sampling problem. One of the first questions to settle is the choice of sampling unit. An entire shipment or lot whose quality is to be estimated is supposedly divisible into identifiable and distinct parts, called sampling units. Every particle of material in the lot must lie in one and only one sampling unit. When material comes in packages, the package is sometimes but not always a natural and economical unit. This is so only if it is possible and not too expensive to designate any particular package for the sample, and to pull it out from the shipment, wherever it may be, for weighing and testing. If the sampling units are to be drawn with equal probabilities, they should be so defined that their weights and other qualities are as nearly alike as is reasonably possible. Sampling with unequal probabilities will not be treated here.

Besides the problem of defining acceptable sampling units, one also requires knowledge regarding the magnitude and stability of the variances between sampling units. It is also necessary to have information regarding the costs of the various operations that occur in the weighing and testing, not only in the regular routine, but as they would be in the conduct of a sampling operation. One of the first steps, therefore, in beginning to lay out a plan of sampling, is to start collecting information on variances and costs.

The sampling of packaged materials; Problem I. — Excellent beginnings on Problems I and II have been made by Williams and Tanner[1]) and co-workers in the Bureau of Customs, and it will be instructive to

[1]) John F. Williams and Louis Tanner, "Statistical methods in the weighing of imported merchandise", read at the meeting of the American Statistical Association in Cleveland, Dec. 1948.

summarize their results.[2]) Wool in bales, raw sugar in sacks, tobacco, burlap, and feathers in bales are imported into the United States in huge shipments. The determination of the weight and average quality of a shipment is necessary for buyer and seller, for statistics on imports and exports, and for assessment of Customs duties. The measurements of weight and quality for the assessment of duty must be carried out by the Bureau of Customs by methods that are recognized in government and industry as efficient, fair, accurate, and economical. These requirements are precisely the aims in modern sample-design.

Single-stage sampling; continuation of Problem I. — In single-stage sampling a sample of m units is chosen and each unit in the sample is tested in full — i.e., not sampled. We shall first lay down some of the general laws for the variances associated with random samples in single-stage sampling.

Let there be M sampling units (packages, articles, bales) numbered from 1 to M inclusive, any one of which may be found and identified unequivocally. It will be assumed that a random sample of m primary units is to be drawn. A convenient way to accomplish this random drawing in practice is to use a table of random numbers. If (e.g.) $M = 1267$ and m is to be 50, one merely reads up or down a column of 4-digit random sampling numbers, discards 0000 and all numbers above 1267, but accepts the first 50 distinct numbers between 0001 and 1267 inclusive.

Before studying the sampling variance of the estimated total weight of the M packages it is desirable to adopt some symbols. For the sake of simplicity in exposition, let weight be the characteristic which is to be measured, and let the term package be used for a sampling unit.

Weights of the M packages of the lot	$x_i \; (i = 1, 2, \ldots, M)$
Weights of the m packages of the sample	$x_i \; (i = 1, 2, \ldots, m)$
Average weight of the M packages of the lot	$\mu = \frac{1}{M} \sum_1^M x_i$
Average weight of the m packages of the sample	$\bar{x} = \frac{1}{m} \sum_1^m x_i$
Variance of the packages in the lot	$\sigma^2 = \frac{1}{M} \sum_1^M (x_i - \mu)^2$
Variance of the packages in the sample	$s^2 = \frac{1}{m} \sum_1^m (x_i - \bar{x})^2$
Total weight of the lot	$M\mu = A$
Total weight of the lot as estimated from the sample	$M\bar{x} = X$
Coefficient of variation, or relative standard error, of the estimate $M\bar{x}$ or of $\bar{x}$ itself	$C_X = C_{\bar{x}}$

[2]) A further paper on the sampling of materials was published by Louis Tanner and W. Edwards Deming, "Some problems in the sampling of bulk materials" (Proceedings of the American Society for Testing Materials, vol. 49, 1949).

If the units of the sample are randomly drawn, then the estimate $M\bar{x}$ or X is a random variable and its coefficient of variation or relative standard error will be given by

$$C^2_X = C^2_{\bar{x}} = \frac{M-m}{M-1} \quad \frac{\sigma^2}{m\,\mu^2} \tag{1}$$

This equation is fundamental and is derived in elementary books on sampling It holds for any distribution of the M weights whose variance is σ^2.

Clearly, the precision increases — i.e., C_X and $C_{\bar{x}}$ decrease — as the sample-size m is made larger. The precision may be controlled as desired by choosing the correct size of sample. Thus, if a standard error of 1% in $\bar{x}$ or X is desired, and if $M = 1267$, $\mu = 300$ kilograms, and $\sigma/\mu = 5\%$, then from Eq. 1,

$$m = \left\{ 1 - \frac{m-1}{M-1} \right\} \left(\frac{\sigma}{\mu\, C_X} \right)^2 \tag{2}$$

This equation can be solved for m without difficulty and the result is $m = 25$. Thus, if C_X or $C_{\bar{x}}$ were to be 1%, the weights of a random sample of 25 packages would be sufficient.

If a smaller error were desired, for example ½ %, the required size of sample would rise to 93.

In practice, if a sample of packages is to be designated for weighing while the lot is being loaded or unloaded, it is customary to weigh every n-th package. In the problem just solved $M = 1267$, $m = 25$, and every 50th package would be weighed, following a random start between 1 and 50. A sample made up of every n-th package is called a patterned or systematic selection. The sampling variance for such a selection will usually be slightly lower than the variance for a random sample, which is given by Eq. 1. On the other hand, there are circumstances in which the sampling variance of a systematic selection is higher than the variance of a random sample. Nothing takes the place of some actual experimentation with the material which is to be sampled.

The cost and the savings that are to be expected from the use of sampling may usually be computed approximately, in advance, on the basis of rough estimates of the variance σ^2 and the cost of handling the material, all based on past experience, of course. The savings will depend upon the relative ease with which packages may be designated for the sample and weighed, and then restored to the lot. When the packages constituting a lot are already piled in a warehouse, it may be expensive to pull out a particular package from the inside or bottom of the pile. Moreover, no particular package can be designated unless the M packages can be numbered 1 to M, somehow or other, nor unless the location of any particular package is known, so that it can be found. Usually costs are much lower and the savings greatest if the packages are weighed as they are loaded or unloaded.

The potential savings to be made by introducing sampling are often great, although in practice ingenuity is sometimes required in order to arrange the work so as to realize these potentialities. In practice the savings often run as high as 70 % of the cost of weighing all M packages, and sometimes the savings are much higher.

An example of single-stage sampling occurs in the weighing of refined sugar as it is imported into the United States. [3]) This commodity arrives in large lots, often containing as many as 43,000 standard-size sacks, weighing approximately 100 lbs. each. The weighing is usually done in drafts of 8 sacks at the warehouse after the damaged sacks, about 1.4 % of the total, have been segregated for separate handling. A study of previous weighings showed that the coefficient of variation σ/μ of the weights of the drafts (8 sacks each) of the sound sacks of refined sugar was stable and averaged only about 0.1 %. From Eq. 2 it is seen that the weighing of but 1 random draft would yield an estimate (X) for the weight of the total shipment with a coefficient of variation of this same magnitude. The weighing of 9 drafts would yield an estimate (X) having a 3-sigma limit of only $\pm$ 0.1 %, meaning that the error of sampling would rarely if ever fall outside $\pm$ 0.1 %. In practice, a sample of 25 drafts might be weighed, as the cost of weighing 25 is negligible compared with the cost of weighing the entire shipment. Moreover, 25 drafts are sufficient to give a fair estimate of σ. It is desirable to make estimates of σ pretty regularly in order to provide continuing knowledge and experience with its magnitude and variability.

Sampling in two stages; continuation of Problem I. — Instead of sampling in a single stage it is sometimes more economical to take the samples of material in two stages, or even in three, especially if the packages are already piled in a warehouse in such manner that it is difficult to extract any particular package. In two stages there will be primary units and secondary units. The primary units might be layers of packages, or sections of layers. In the sampling of baled wool (vide infra) to estimate the clean-content of a lot of bales, the primary units will be bales, and the secondary units will be cores which may be cut from the bales. The primary units should be as nearly the same size and quality as possible, and likewise for the secondary units. The argument for this statement will be seen from Eq. 3. If the primary units are very unequal, C_b^2 will be large, and if the secondary units are very unequal, C_w^2 will be large. A large value of either one of these quantities will increase the size of sample that is required to reach a desired precision.

It will be supposed that the primary units may be numbered and identified 1 to M inclusive. It will be supposed, furthermore, that secondary units, perhaps in the form of packages, may be numbered and identified 1 to N_i within the i-th primary unit of the sample. The problem of sampling is to find how many primary units to draw, and how many secondary units from each primary unit of the sample should be drawn and weighed or otherwise tested in order to meet the requirements of precision at the lowest possible cost.

It will be assumed that primary and secondary units will both be drawn at random, and that a table of random numbers may be used for selecting the samples, although, as in single-stage sampling, a systematic selection may be used instead of a random selection.

For two-stage sampling it will be necessary to adopt some further notation.

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[3]) The remainder of this paragraph is quoted almost verbatim from the paper by Tanner and Deming, cited earlier.

It will be assumed here that $N_1 = N_2 = \ldots = \overline{N}$, both for simplicity and because this is often approximately the actual condition. [4])

k_1	the cost of preparing a primary unit for sampling
k_2	the cost of taking a secondary unit from a primary unit
M	the number of primary units in the lot
m	the number of primary units in the sample
$\overline{N}$	the number of secondary units in each primary unit
$\overline{n}$	the number of secondary units to be drawn from each primary unit in the sample
σ_b^2	the variance between the M primary units of the lot [5])
$C_b = \sigma_b/\mu$	the coefficient of variation between the M primary units
σ_w^2	the average variance between the secondary units within the primary units of the lot
$C_w = \sigma_w/\mu$	the average coefficient of variation between the secondary units
$\overline{x}$	the average weight of the $m\overline{n}$ test-portions and the estimated average weight of the entire lot
$X = M\overline{N}\overline{x}$	the estimated weight of the entire lot

The coefficient of variation C_X of the estimated total weight of the lot, and the coefficient of variation $C_{\overline{x}}$ of the average weight or other quality per secondary unit, are dependent upon M, m, C_b and C_w by the relation

$$C^2_X = C^2_{\overline{x}} = \frac{M-m}{M-1}\frac{C_b^2}{m} + \frac{\overline{N}-\overline{n}}{\overline{N}-1}\frac{C_w^2}{m\overline{n}} \tag{3}$$

To find σ_X it is only necessary to multiply C_X by $M\overline{N}\mu$, and to find $\sigma_{\overline{x}}$ it is only necessary to multiply $C_{\overline{x}}$ by μ. Eq. 3 is easy to remember and it expresses both C_X and $C_{\overline{x}}$.

The total cost of making the $m\overline{n}$ tests can be represented approximately by the equation

$$K = m k_1 + m\,\overline{n}\,k_2 \tag{4}$$

The best sampling plan for any particular set of costs and variances (k_1, k_2, σ_b^2, σ_w^2) is that which produces a desired coefficient of variation C_X or $C_{\overline{x}}$ at the same time minimizing the cost, K. Or, the best sampling plan is that which extracts the most information possible for an allowable cost, K. It may be proved that these conditions are met pretty closely if [6]) [7])

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[4]) If the numbers N_1, N_2, ... are not all equal, or approximately so, Eq. 3 will be more complicated, and the reader may wish to consult a treatise on sampling for more complete theory.

[5]) For more detailed and more general definitions of these variances, and for derivations of Eqs. 3ff, the reader may wish to consult a treatise on sampling.

[6]) Eq. 5 seems to have been published almost simultaneously by L. H. C. Tippett, "Methods of Statistics" (Williams & Norgate, 1931), p. 177, and by Walter A. Shewhart, "The Economic Control of Quality of Manufactured Product" (Van Nostrand, 1931), p. 389.

[7]) See next page.

$$\bar{n} = \frac{\sigma_w}{\sigma_b} \sqrt{\frac{k_1}{k_2}} = \frac{C_w}{C_b} \sqrt{\frac{k_1}{k_2}} \tag{5}$$

and

$$m = \frac{M \bar{n} C_b^2 + \dfrac{\bar{N} - \bar{n}}{\bar{N} - 1} (M - 1)\, C_w^2}{(M - 1)\, \bar{n}\, C_{\bar{x}}^2 + M \bar{n}\, C_w^2} \tag{6}$$

Eq. 5 is to be solved for $\bar{n}$ in order to learn how many secondary units should be drawn from each primary unit, and then Eq. 6 is to be solved for m in order to learn how many primary units should be drawn from the lot. It is interesting to note that the optimum value of $\bar{n}$ (number of secondary units per primary unit) does not depend on m nor on the desired precision, C_X or $C_{\bar{x}}$.

Obviously the solutions for $\bar{n}$ and m require knowledge of the variances σ_b^2 and σ_w^2 and of the costs k_1 and k_2. $C_{\bar{x}}$ in Eq. 6 is the desired coefficient of variation of the estimate of either $\bar{x}$ or X; it might be (e. g.) ½%. It is important to note, however, (*a*) that $\bar{n}$ is determined if only the ratios $\sigma_b : \sigma_w$ and $k_1 : k_2$ are known; and (*b*) that no great accuracy is needed in these ratios, particularly in $k_1 : k_2$ because it occurs under the square root. Even approximate values of these ratios will easily distinguish between a very costly sampling plan and one whose cost (K) is within 5% of the cost of the optimum plan, which may only be devised with the aid of firm values of the variances and costs. It is remarkable how much saving can be made with very meagre information.

An example may be helpful at this point. [8]) Raw wool appears is commerce in lots of various sizes, averaging about 100 bales, but running sometimes as high as 3000. The bales range in weight from 200 lbs. to 1200 lbs. but are fairly uniform from any one source. The wool contains grease and other impurities, the percentage of which may vary as much as 30 %, both between bales in a lot and between portions within a bale. The need of a reliable method for determining the percentage of clean wool without processing all or a large fraction of the lot has long been recognized. The major obstacle to the development of such a method has until recently been the unreliability of the only type of small sample that could be taken, usually a few handfuls of fleeces selected on the basis of expert judgment from a few bales in the hope that they would be representative. This particular difficulty has been eliminated by the development of a practicable sampling tool that can penetrate into a bale of wool at any position designated at random and can remove therefrom a relatively small core. It is important to calculate by the equations given earlier how many such cores should be taken per bale, and from how many bales, in order to estimate the proportion of clean wool in the lot with the greatest economy.

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[7]) If σ_w^2 is not small compared with $\bar{N}\sigma_b^2$, Eq. 5 should be replaced by

$$\bar{n} = \frac{\sigma_w}{\sigma_b} \sqrt{\frac{k_1}{k_2} \frac{1}{1 + \sigma_w^2 / \bar{N} \sigma_b^2}} = \frac{C_w}{C_b} \sqrt{\frac{k_1}{k_2} \frac{1}{1 + C_w^2 / \bar{N} C_b^2}}$$

[8]) The rest of this section is quoted almost verbatim from the paper by Tanner and Deming, cited earlier.

In the sampling of wool the primary unit is the bale; the secondary unit is the core. For any possible value of $\bar{n}$ (such as $\bar{n} = 1$, or 2, or 4, etc.), the corresponding value of m is found from Eq. 6. A pair of values, $\bar{n}$ and m, so determined, are said to constitute an optimum sampling plan — a plan that will yield the coefficient of variation $C_{\bar{x}}$ with the greatest economy. Different sampling plans may be devised, all yielding the same precision, but costing different amounts, under the assumptions made. Naturally, that plan is best which costs the least.

Systematic studies have been made of the magnitudes and stability of σ_w and σ_b for many commercial lots of most types and grades of wool from different sources throughout the world. As was to be expected, no single pair of values was found to be valid for all different kinds of wool. However, it was found that, when grouped in a relatively small number of broad classifications according to origin, type of wool and type of bale, the variances were fairly stable and could provide useful estimates for devising sampling plans. Some typical values of variances are shown in Table I.

By using these estimates of σ_w and σ_b in Eq. 6, sampling plans may be prepared which will give for each class of wool the number of bales (m) to be sampled for various lot sizes (M) and for various numbers of cores ($\bar{n}$) per bale, for any required level of precision. All of the four sampling plans illustrated in Table II will give the same precision, but their costs will be different.

Table I.

Classification of packaged greasy apparel wools for sampling purposes.

Type of wool	σ_w	σ_b
Argentina	2.5	2.5
Australia	1.5	4.0
Chile	2.0	5.0
Uruguay	2.0	1.5
Domestic (U. S.)	4.5	2.0

Table II.

Sampling schedule showing the number of bales to be sampled from a lot for which $\sigma_w = \sigma_b = 2.5\%$, for a standard error of $1/2$ ..

M Number of bales in the lot	m Number of bales to be in the sample			
	$\bar{n} = 1$	$\bar{n} = 2$	$\bar{n} = 4$	$\bar{n} = 6$
25	25	19	16	15
50	34	25	21	20
75	38	29	24	22
100	40	30	25	24
150	43	33	27	25
200	45	34	28	26
300	47	35	29	27
500	48	36	30	28
1 000	49	37	31	29

There are two operations in obtaining a core of wool: first, putting the bale into position, and second, boring it. Each operation has its own cost, [9] k_1 and k_2 respectively, and their ratio may vary widely depending on the conditions existing. If sampling takes place at the time the individual bales are being weighed or moved to storage, there is virtually no additional cost for putting the sample bales into position, and $k_1 : k_2$ might be, for example, 1.0. On the other hand, if the bales are already stored in piles in a warehouse, and if the piles must be disturbed and moved in order to draw a sample of bales, the ratio $k_1 : k_2$ of costs might easily exceed 25.

The usefulness of this theory has been proved in actual practice. One testing organization is regularly called upon to sample lots of domestic wool stored in warehouses. For this wool $\sigma_w : \sigma_b$ is about 2.25, and under the conditions of sampling, $k_1 : k_2$ is approximately 20. Accordingly, the best sampling plan to be followed there is to take 10 cores from each bale ($\overline{n} = 10$), and to determine the number of bales from Eq. 6. A second organization is able to sample its imported carpet wool as it is being carried by truck into the mill. In this case $k_1 : k_2$ is about 1, and $\sigma_w : \sigma_b$ also averages about 1, so that the sampling plan used is one core per sampled bale ($\overline{n} = 1$). The two plans differ considerably for the same precision, yet each is the most economical plan that is possible under the different conditions faced by the two organizations.

T h e s a m p l i n g o f p a c k a g e d m a t e r i a l s ; P r o b l e m II. — In this problem there will be a weight or other quality Y_i which has been determined previously for each unit, $i = 1, 2, \ldots, M$. Suppose that a random sample of m packages is drawn at random from the lot, and that a re-determination X_i is made for each of the packages in the sample. Let

$$f = \frac{\sum_{1}^{m} X_i}{\sum_{1}^{m} Y_i} \tag{7}$$

Define also

$$X = \sum_{1}^{M} X_i \quad \text{(Assumed not known in practice)} \qquad Y = \sum_{1}^{M} Y_i \quad \text{(Assumed known in practice)} \tag{8}$$

A value of f, once determined by weighing m sample bales from a lot, is then used as a conversion factor to calculate an estimate X' of the total weight of the lot by converting Y from the old weights to the new weights by the relation

$$X' = f\,Y \tag{9}$$

The coefficient of variation of f in repeated samples will be approximated by the equation [10]

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[9] If each secondary unit is tested individually, k_2 should include the unit cost of testing. With wool, the cores are compounded, and a single test is performed on the composite sample, wherefore the cost of testing is independent of $\overline{n}$ and m, and is not included in k_2.

[10] The derivation of this equation is in Frank Yates's "Sampling Methods for Censuses and Surveys" (Chas. Griffin & Co., 1949). It is derived in the notation of this paper in Deming's "Some Theory of Sampling" (John Wiley, 1950), Ch. 5.

$$C^2_f = \frac{M-m}{M} \quad \frac{1}{m(m-1)} \quad \sum_1^m \left[\frac{X_i - f\,Y_i}{\bar{X}} \right]^2 \tag{10}$$

wherein

$$\bar{X} = \frac{1}{m} \sum_1^m X_i \tag{11}$$

As Y in Eq. 9 is a constant, it follows that

$$C^2_{X'} = C^2_f \tag{12}$$

The factor $\frac{1}{m-1} \sum_1^m \left[\frac{X_i - f\,Y_i}{\bar{X}} \right]^2$ in Eq. 10 is an estimate of the coefficient of variation of the ratio $X_i : Y_i$ multiplied by $M/(M-1)$. Clearly, if the correlation between X_i and Y_i is high, the coefficients of variation C_X and C_f will be small and the precision of the estimates X and f will be high, even with small samples.

Because the fraction f in Eq. 7 is used as a calibration factor in Eq. 9, this technique, making use of the information already contained in the Y_i, is sometimes called a *calibration* technique.

An example of the use of the calibration technique is seen in the weighing of imported tobacco, of which 100,000 000 lbs. are imported annually into the United States. This commodity arrives in lots of several hundred or thousand bales of various sizes ranging in weight from 10 to 300 pounds, according to source.

The weights of the individual bales have usually been determined some time before they were loaded on to the ship in some foreign port. The total weight of a lot, so determined, however, is not acceptable for assessment of duty, nor usually for commerce either. Re-weighing is therefore considered to be necessary. However, it may only be necessary to weigh a few bales.

Previous practice in the Bureau of Customs was to weigh every bale individually — a slow and expensive procedure. In developing a sampling plan it was found that the coefficient of variation of the weights of the bales averages about 3.5 % and ranges from 1.5 % to 9.4 %. The sampling methods of Problem I are therefore not satisfactory because so large a sample would be required for any reasonable degree of accuracy that no material saving in weighing would be made.

However, by using the calibration technique, it is not the nearconstancy of the weights Y_i which is important, but rather, the nearconstancy of the ratio $X_i : Y_i$. If Y_i denotes the weight of a bale as determined before it was put on board the ship, and if X_i denotes the result of re-weighing the bale as it is landed, it has been found that the ratio $X_i : Y_i$ is very nearly the same for all bales. Eqs. 7—12 can therefore be applied with considerable satisfaction. The coefficient of variation of $X_i : Y_i$ has been found to be fairly stable, averaging about 0.7% except for bales of Greek tobacco for which the average was about 2.5%. On the basis of these values, Eq. 4 shows that a sample size of $m = 100$ bales from shipments of most cigarette tobacco (900 for lots of Greek tobacco) is adequate to achieve a precision of $\pm$ 0.25%, a level considered satisfactory in protecting both the government and the importer against an appreciable overcharge in duty on any one importation.

On the average, these sample sizes represent savings of from 75 to 95 % in the number of weighings. Similar savings in many other types of problems can be made.

S o m e r e m a r k s o n P r o b l e m III. — This is the problem of estimating (e.g.) the ash-content or B.t.u. content per ton of a pile of coal or a carload or shipload of coal. It should be made clear that no satisfactory statistical procedure has yet been devised, although preliminary investigations indicate hope of an early solution to the main obstacle, which is at present the difficulty of dividing the material into unequivocally identifiable units that can be drawn from the lot in a routine operation. One might vainly specify, for example, that a pile or car of coal is to be sampled by extracting certain cubes of parallelepipeds, defined as located so many feet from the top and having certain other spacial coordinates. It would be impossible, though, as a regular routine operation, for a worker to draw into the sample the contents of any sampling unit and to exclude all material not intended for this unit. Under existing circumstances such a material fails to meet the fundamental criterion of a unique probability for every particle.

The sampling of materials that fall into Problem III is thus by definition not open to the methods that are used for packaged materials falling into Problems I and II. When samples are drawn from any material that is not readily divisible into identifiable units, the drawing must too often depend on human judgment. Such dependence, experience demonstrates, is hazardous, as severe biases often develop. As a matter of fact, biases are the rule, not the exception. No statistical measure of precision can be calculated for a judgment-selection. It is thus highly important that the statistical problems of sampling coal and other such materials be solved, because of the magnitudes of the commercial transactions involved.

The sampling of materials that fall into Problem III is not hopeless, but calls for ingenuity and a new attack. If such a material can be sampled off a conveyor belt at specified intervals, it is, in effect, divisible into unequivocal sampling units, no matter whence it came nor how it is piled up afterward. When sampled off a conveyor belt a material is amenable to the methods of Problem I.

S o m e r e m a r k s o n P r o b l e m IV. — The use of modern procedures for the determination of the average physical condition of the various classes of property comprising a telephone, telegraph, railway, or other utility company has only recently commenced. As stated in the opening section, this problem is important in judiciary cases involving reserve and equitable rates for service. At other times, the management of a utility may wish to know the state of depreciation of the various classes of the property, as in making estimates of the type and extent of maintenance and repairs that will be required by the system over the next 5 or 10 years.

This type of problem, from the viewpoint of sampling, is often one of the most satisfying and simplest, especially when, as is usually true, the records, maps, and lists maintained by the company show the locations and descriptions of all of the various items of the property in minute detail. In some cities of this country, such records are required by law: in many cities, the records are kept anyhow as being necessary to good management.

It is the intent of the author to discuss here a small segment of this problem, in the hope that a few remarks may find extension, modification, and application in other related problems which exist in abundance the world over.

Suppose that the property of a large public utility company is to be appraised to determine its over-all physical condition within some narrow range of precision. The property consists of (e.g.) 14 main accounts — poles and the aerial equipment maintained thereon, manholes and underground cable and conduit therein, meters, terminals, switches, transformers, etc. Samples of items in each account are to be drawn and examined by skilled inspectors, thoroughly trained, and the physical condition of each item inspected is to be rated as A, B, C, D, or E, which might correspond to 95, 80, 65 etc., the exact definitions for which need not concern us here. Let w_i be the proportional book value of the i-th account when new; then $\sum w_i = 1$. If every item in every account had an equal chance with every other item of coming into the sample, and if x_i is the average grade of the items inspected in the i-th account, then

$$x = \sum w_i x_i \tag{13}$$

will be an unbiased estimate of the weighted average condition of the entire property comprised in all the accounts.

The problem is to determine the sample-sizes n_i for maximum economy. Obviously, the samples from important accounts like aerial plant (poles and cable plus attachments) should be bigger than the samples from less valuable accounts like terminals and house-cable.

The determination of the optimum value of n_i is not difficult. Let n_i be the number of items to be inspected in the i-th account, and let k_i be the cost of inspecting one item therein, including travel and tabulation. Then the total cost of inspection in all accounts will be

$$K = \sum n_i k_i \tag{14}$$

Let σ_i be the standard deviation of the numerical grades in the i-th account. Then as the samples in the different accounts are entirely independent,

$$\sigma_x^2 = \sum \left[\frac{w_i \sigma_i}{\sqrt{n_i}} \right]^2 \tag{15}$$

The most economical plan would be to adjust the sample sizes (n_i) so that σ_x^2 is made as small as possible for a given allowable cost, K. It may be shown that this condition leads to the equation

$$n_i = \frac{w_i \sigma_i}{\lambda \sqrt{k_i}} \tag{16}$$

the derivation for which will not be given here. λ is a constant for all classes of the property, and its value depends on the desired value of σ_x. The factor $\sigma_i / \sqrt{k_i}$ is usually highly variable from one account to another, and the sample should be adjusted up or down in each account in accordance with the values of $\sigma_i / \sqrt{k_i}$ that are likely to be encountered.

Advance estimates of σ_i are not difficult to make. The condition of any item must lie between a and b, where $0 < a < b \leq 100$, a being the lower limit of physical condition, and b the upper limit. For some classes of property, such as telephone poles, cables, and terminals, a may be near 0 and b near 90.

For certain types of apparatus that must be kept in firstclass condition, such as switches in dial switching equipment, or teleprinters as another example, a is probably not less than 70, and b may be as high as 95 or 98. If the condition of such pieces of apparatus is permitted to go below 70, the apparatus will fail to perform its function.

In practice, in the absence of other information, it is sufficient to assume that the physical conditions of the items of any particular class of property are distributed in a rectangle between a and b. An advance estimate of σ_i for that class is then found by setting

$$\sigma_i^2 = \frac{1}{12}(b - a)^2 \tag{17}$$

With the aid of engineers who are thoroughly familiar with the property, excellent estimates of σ_i may be made in advance by the use of Eq. 17, whence an economical and adequate sample may be designed. The biggest possible value of σ_i may be allowed for by setting $a = 0$ and $b = 100\%$, the result of which gives σ_i equal to 29%. A random sample of $n_i = 400$ would then give a maximum standard error in x_i equal to about $1\frac{1}{2}\%$. (Actually, the systematic samples about to be described will usually do a little better than is indicated by Eq. 17)

Useful estimates of the costs k_i can almost always also be made in advance. Then for two classes of property, Class 1 and Class 2, Eq. 16 gives

$$\frac{n_1}{n_2} = \frac{w_1 \sigma_1}{w_2 \sigma_2} \sqrt{\frac{k_2}{k_1}} \tag{18}$$

This equation is fundamental. Again, as in the discussion of Problem II, even rough values of $\sigma_1 : \sigma_2$ and of $k_1 : k_2$ will guide the statistician toward an economical sample. The weights w_1 and w_2 are supposedly obtainable from the company's accounting records, and can hardly be classed as a part of the sampling problem, because they would require determination in any case, no matter how small or how large the samples may be — in fact, they would be the same even if the samples were total (100%).

It is important to note that the precision that is actually attained can be appraised accurately and easily, as explained in the next section. This appraisal is entirely independent of the judgment that went into the advance estimates of the variances and costs.

Appraisal of the precision attained. — The procedure recommended here depends upon the extremely simple idea of repeating the sampling procedure 10 times and comparing the results. I first used it at the suggestion of Professor John W. Tukey of Princeton. For example, a telephone company owns equipment on approximately 973,000 poles. Suppose that a total sample of 1500 poles is desired. The total sample is to be composed of 10 independent subsamples, each a 10th of the total in size. The first step is to determine the proper counting interval for the 10 subsamples of 150 poles each. The counting interval for the subsamples will be

$$\frac{973{,}000}{150} = 6487 \tag{19}$$

As any advance estimated total, such as 973,000, may be in error, it is well to reduce this number [11]) to 6387. The 10 independent systematic subsamples will be obtained by finding 10 random starts between 1 and 6387. One such sampling table is reproduced in Table III.

Table III. *)

An example of a table of sampling numbers giving 10 independent subsamples.

Counting interval, 6 387.

Subsample No.	Random starts	Additional sampling numbers formed by adding 6,387 successively								
1	4,746	11,133	17.520	23,907	30,294	36,681	43,068	49,455	55.842	62,229
2	5,761	12,148	18,535	24,922	31,309	37,696	44,083	50,470	56,857	63,244
3	0,830	7,217	13,604	19,991	26,378	32,765	39,152	45,539	51,926	58,313
4	5,550	11,937	18,324	24,711	31,098	37,485	43,872	50,259	56,646	63,033
5	1,603	7,990	14,377	20,764	27.151	33.538	39,925	46,312	52,699	59,086
6	2,617	9,004	15,391	21,778	28,165	34,552	40,939	47,326	53,713	60,100
7	1,288	7,675	14,062	20,449	26,836	33,223	39,610	45,997	52,384	58,771
8	6,204	12,591	18,978	25,265	31,752	38,139	44,526	50,913	57,300	63,687
9	0,131	6,518	12,905	19,292	25,679	32,066	38.453	44,840	51,227	57,614
10	2,523	8,910	15,297	21.684	28,071	34.458	40,845	47,232	53,619	60,006

*) This table was actually used by the Illinois Bell Telephone Company for drawing a sample of poles and attached aerial equipment.

Every sampling number in the table of sampling numbers will draw one item and any associated equipment into the sample. Thus, Table III might refer specifically to telephone poles. Any sampling number in the table would then bring in a telephone pole plus all the cable, wire, cross-arms, terminals, and other equipment associated with that pole. All this equipment would be inspected when the pole is inspected; it is not necessary to draw new samples of cable, wire, cross-arms, terminals.

However, the physical conditions of these items are correlated and they are best kept as a group to form the "aerial account". Each subsample of 150 poles, when inspected, yields an estimate of the weighted average physical condition of this aerial account. Let $x_1^{(1)}, x_1^{(2)}, \dots, x_1^{(10)}$ be 10 results so obtained, and let

$$x_1 = \frac{1}{10} \sum_{1}^{10} x_1^{(i)} \tag{20}$$

An estimate of the variance of the average x_1 may then be computed as

$$\hat{\sigma}_1^2 = \frac{1}{10} \, \frac{1}{10-1} \sum_{1}^{10} \left[x_1^{(i)} - x_1 \right]^2 \tag{21}$$

[11]) In the sampling of manufactured equipment it is advisable never to use a counting interval ending in 0, 5, or an even number, because of possible periodicities. If the division of 973,000 by 150 had not given a quotient terminating in 7, an arbitrary adjustment would have been made to force the counting interval to terminate in 1, 3, 7, or 9.

Thus, in an actual example, the results of 10 subsamples (each of 150 poles and attachments) were as follows:

67.5 %	68.4
65.9	69.6
67.1	69.4
67.9	69.0
70.8	68.0

whence the average result for this account was 68.4 %, with an estimated standard error of 0.44 % as computed by Eq. 21. As each $x_1^{(i)}$ will in practice be the average of some sizable number of inspections (possibly 8, 10, 15, or more), Fisher's t-table may be used for finding any desired probability limits for the physical condition of this particular account. Thus, in this instance, the odds are 99 : 1 that the value $\bar{x} - .44\, t_{.01} = 68.4 - .44 \times 2.82 = 67.2$ does not fall below the result that would have been obtained by examining all the poles belonging to the company, were it possible to carry out such a huge task with the same care and skill as was expended on the sample.

In a similar manner the variances of the other accounts may also be estimated, and all the values of x_i may be combined to find the estimated average overall physical condition x of the entire property, x being defined in Eq. 13. The estimated variance of x will be

$$\sigma_x^2 = \sum (w_i \hat{\sigma}_i)^2 \tag{22}$$

in which summation the subscript i runs over all the accounts that were sampled independently. In practice, the samples may be designed in advance so that σ_x for the whole property will have a value somewhere between 0.2 to 0.3 %. Any attempt at greater refinement is wasteful and illusory, in view of (a) the difficulties of defining and measuring the physical condition of a piece of property, and (b) the fact that the different inspectors, no matter how expert, can not always agree with each other or even with themselves. Still smaller samples, giving σ_x as high as 0.5 % or even 1 % will contain a wealth of information concerning the property.

Measurement of the human error. — It is a mistake to concentrate any sampling plan purely on the measurement and control of the errors of sampling, and the same statement holds for the interpretation of data derived from sampling. There are other errors, mainly centered around the arbitrariness of definitions and the variability of performance of both man and machine. These other errors are present whether the materials are examined in total or by a very small sample, and they may therefore be called nonsampling errors. These nonsampling errors are as important in the sampling of physical materials as they are in social and economic surveys. What should be aimed at in designing a sample of any material is to strike a balance between the errors of sampling and the errors and variability arising from nonsampling sources.

A list of the nonsampling errors that occur in social and economic surveys was published by the author [12]) in 1944. A list of the nonsampling errors

[12]) W. Edwards Deming, "On errors in surveys" (American Sociological Review, vol. IX, 1944, pp. 359—369); "Sobre errors en las investigaciones" (Estadistica, vol. VI, 1948, pp. 493—504; vol. VII, 1949, pp. 84—91). This list is expanded in the author's "Some Theory of Sampling" (John Wiley & Sons, 1950), Ch. 2.

that are encountered in the measurement of physical materials would be remarkably parallel to this list just cited. The chief difference is that repeated physical observations do not always alter the material, whereas repeated interviews of human beings probably do affect the responses to most questions.

It is wasteful to refine the errors of sampling to levels far below the errors arising from other sources, hence knowledge of the magnitude of the human errors of measurement is necessary as a guide for achieving economical and effective sample design.

Success in reducing the nonsampling errors depends to a large extent on the selection, training, and supervision of the men who do the actual work of inspection and measurement. There are many schemes that have been devised to achieve uniformity and accuracy of human performance. One device will be mentioned here. In the training of the inspectors who will inspect the samples of the property of a utility company, and during the actual inspection, it is possible and advisable to carry out experiments in which all the men inspect a number of pieces of equipment, the aim of the experiment being to observe whether any inspectors are grading the material higher or lower than the others, and to take corrective action if necessary. Of course, exact agreement is not to be expected.

One set of records which was obtained in an experiment of this nature is shown in Table IV. Each inspector recorded his judgment, A, B, C, D, or E, for each item of a particular type of apparatus, and the results were assembled in Table IV. The inspectors had received five days' training when this experiment was carried out. Grade A corresponds to 92 %, new or nearly new; Grade E corresponds to 13 % worn out. Actually, the grades were very carefully defined with written definitions, and the inspectors were all skilled telephone engineers, thoroughly familiar with the apparatus. If any inspector had given grades consistently higher or lower than the others, a study of the table would probably have detected it.

Mahalanobis [13]), through the use of interpenetrating samples, measured the combined effect of errors of sampling and the differences in human performance, and compared these effects with the average variability in the results that were observed by individual interviewers. In this way he measured the ability of interviewers to reproduce their own work, and measured the differences between interviewers.

The plan followed here for measuring simultaneously the differences between inspectors and the errors arising from sampling was suggested to the author by Dr. W. J. Youden of the National Bureau of Standards in Washington. Let us return to the 10 subsamples that were mentioned earlier for a sample of the aerial property of a telephone system (Table III). Each subsample is to contain 150 poles plus the equipment associated with each pole. Suppose that there will be 5 inspection-crews, 2 men per crew, one of them being an expert on this type of equipment, the other man his assistant. A sampling plan in which there are $5 \times 10 = 50$ valid subsamples of the property would permit each crew to work on 10 different subsamples. The Youden plan assigns each crew to 10 valid independent subsamples of the property. For the aerial

[13]) P. C. Mahalanobis, "On large-scale sample surveys" (Phil. Trans. Royal Soc., vol. 231 B, 1944, pp. 329—451; pp. 407—410 in particular). Same author, "Recent experiments in statistical sampling in the Indian Statistical Institute" (J. Royal Stat. Soc., vol. CIX, 1946, pp. 325—378).

property, for example, the crews were assigned to the poles in rotation according to the scheme shown in Table V, which when superimposed over Table IV shows which crew is to be assigned to inspect any particular pole plus its associated equipment. Obviously, by this arrangement every crew inspects a systematic sample from every one of the 10 subsamples of the aerial property, and the inpectors' samples will be very nearly equal in size.

Table IV.

Results from 8 inspectors who recorded their judgments of the physical conditions of 16 pieces of cable strand, complete with rings.

These results were obtained after two days' traning.
All discrepancies were thoroughly discussed and resolved.

Item. No.	Inspector							
	1	2	3	4	5	6	7	8
1	A	A	A	A	A	A	A	A
2	A	A	A	A	B	A	A	B
3	E	E	E	E	E	E	D	E
4	E	E	E	E	E	E	E	E
5	E	E	E	E	E	E	E	E
6	B	B	A	B	B	A	B	B
7	D	D	D	D	D	D	D	D
8	A	A	A	A	A	A	A	A
9	E	D	E	E	E	D	D	E
10	A	A	A	A	B	A	A	B
11	A	A	A	A	A	A	A	A
12	D	D	D	D	D	D	D	D
13	E	E	E	E	E	E	D	E
14	E	E	E	E	E	E	E	E
15	E	E	E	E	E	E	E	E
16	E	E	D	E	E	D	D	E

Table V.

Youden's scheme for assigning crews.

This table is to be superimposed over Table IV
to see which crew is to be assigned to any particular pole.

Subsample No.	Crew										
1	1	2	3	4	5	1	2	3	4	5	1	Etc.
2	2	3	4	5	1	2	3	4	5	1	2	Etc.
3	3	4	5	1	2	3	4	5	1	2	3	Etc.
4	4	5	1	2	3	4	5	1	2	3	4	Etc.
5	5	1	2	3	4	5	1	2	3	4	5	Etc.
6	1	2	3	4	5	1	2	3	4	5	1	Etc.
7	2	3	4	5	1	2	3	4	5	1	2	Etc.
8	3	4	5	1	2	3	4	5	1	2	3	Etc.
9	4	5	1	2	3	4	5	1	2	3	4	Etc.
10	5	1	2	3	4	5	1	2	3	4	5	Etc.

In practice, there may be more or less than 5 crews for any class of property, but the same Youden rotation may be used for any number of crews and any number of subsamples.

One set of results [14]) which was obtained in an investigation of the aerial property of a telephone system in America is shown in Table VI. In this compilation of results, the 10 averages in the extreme right-hand column give a measure of the pure sampling error, as the differences between crews has had a chance to cancel out. Likewise, the 8 averages along the bottom give a measure of the variability between the crews, as any differences between subsamples has had a chance to cancel out.

Table VI.

Results from inspecting samples of treated poles.

Average per cent condition by crew by subsample.

Subsample	Crew								Average
	1	2	3	4	5	6	7	8	
1	58.9	58.5	62.9	50.5	57.1	63.3	55.5	80.7	60.92
2	67.3	59.9	62.9	63.5	64.0	56.1	75.5	53.9	62.89
3	58.1	82.1	60.3	54.1	62.4	72.6	66.9	61.7	64.77
4	73.3	77.1	58.3	62.6	65.6	73.8	68.0	65.9	68.07
5	55.1	57.2	73.3	68.1	65.1	71.8	57.5	59.2	63.41
6	63.0	62.5	59.1	68.9	77.1	63.4	59.4	65.4	64.85
7	61.0	59.8	63.0	56.7	60.3	74.7	66.4	61.8	62.96
8	59.8	57.7	57.2	58.8	78.7	54.1	59.3	60.8	60.80
9	61.2	57.8	67.8	57.6	65.8	61.3	57.3	83.3	64.01
10	77.7	59.2	66.8	73.2	70.4	66.4	67.0	62.6	67.91
Average	63.54	63.18	63.16	61.40	66.65	65.75	63.28	65.53	64.06

Here there were 10 subsamples and 8 crews, each of which inspected a sample of from 15 to 19 treated poles in each subsample. It is to be noted that this number is sufficient to validate the assumption of normality on which Fisher's tables for the analysis of variance are based. The analysis of variance appears in Table VII, whence it may be concluded [15]) [16]) (a) that there is no indication of uncontrolled variability between the means of the crews; (b) that there is no evidence of excessive variability in the repeated judgments of any one crew; also (c) that the sample is large enough.

[14]) It is a pleasure to record my appreciation of the occasion to work in Chicago with Mr. Harlow A. Coxe of the Illinois Bell Telephone Company in the derivation of these results, and to thank this Company for the use of Tables III—VII.

[15]) I wish to record here my deep appreciation for the aid of Mr. Howard L. Jones of the Illinois Bell Telephone Company, Chicago, both for furnishing the calculations appearing in Table VII and for his generous assistance in the interpretations of the variances occurring therein.

[16]) It should be explained that although the 80 samples in Table VI were all systematic, the variances in Table VII are interpreted as if the samples were random. If there is ever any doubt about the validity of applying the usual theory of the analysis of variance to systematic samples, care should be taken in advance to use a random selection. The systematic rotation of crews, as used here, has great advantages because of its simplicity, and I believe that for this material and for the sampling intervals that were used here, the systematic assignment will give closely the same results as if the assignments were random.

Conclusion (a) follows from the fact that the estimate (.380) of $\sigma^2_{\bar{x}}$, computed from the variance between the means of the crews is actually smaller than the estimate (.617) computed from the variance between the means of the subsamples and smaller also than the estimate (.689) computed from the residual variance. As any real difference between the means of the crews would, on the average, cause the estimate of $\sigma^2_{\bar{x}}$ made from the variance between crews to be larger than the estimate made from the variance between subsamples, it may be concluded that the table shows no evidence of variability between the means of the crews. Conclusion (b) is drawn from the fact that the range of the estimates of $\sigma^2_{\bar{x}}$ based on the variances between subsamples for the 8 different crews (bottom panel of Table VII) is slightly less than the range that would be expected if all the inspection had been done by the same crew.

Conclusion (c) follows from the fact that the estimate of $\sigma^2_{\bar{x}}$ made from either the means of the subsamples or from the residual variance is already small enough.

Table VII.

Variances computed from Table VI. *)

Component	Sum of squares	Degrees of freedom	$\hat{\sigma}^2$	$\hat{\sigma}^2_{\bar{x}} = \frac{1}{80}\hat{\sigma}^2$
Total variance	4129.27	79	52.27	0.653
Variance between the means of crews	212.64	7	30.38	0.380
Variance between the means of subsamples	444.43	9	49.38	0.617
Residual variance	3472.20	63	55.11	0.689
Variance between crews				
In subsample 1	563.31	7	80.47	1.006
2	315.93	7	45.13	0.564
3	559.54	7	79.93	0.999
4	277.92	7	39.70	0.496
5	353.33	7	50.48	0.631
6	240.58	7	34.37	0.430
7	211.10	7	30.16	0.377
8	395.12	7	56.45	0.706
9	529.59	7	75.66	0.946
10	238.43	7	34.06	0.426
Variance between subsamples				
By crew 1	458.46	9	50.94	0.637
2	706.86	9	78.54	0.982
3	221.56	9	24.62	0.308
4	461.62	9	51.29	0.641
5	429.30	9	47.70	0.596
6	489.03	9	54.34	0.679
7	368.88	9	40.99	0.512
8	780.92	9	86.77	1.085

*) $\hat{\sigma}^2$ here denotes an estimate of the variance σ^2 of a universe of values whence it is assumed that each of the 80 cells in Table VI is drawn. $\hat{\sigma}^2_{\bar{x}}$ is the estimated variance of $\bar{x}$.

The advantages of having measures of the sampling error and of the human performance, separate and distinct, seem to be worth consideration in spite of the extra cost of assigning to each crew 10 valid samples of the universe of material that is to be examined. It would of course be cheaper to assign each crew to some convenient area, but the results would not then permit separation of the sampling errors from the human variability.

Résumé: Le sondage est appliqué de diverses manières pour déterminer le poids et la qualité des matières industrielles. L'application de la théorie du sondage donne, rapidement et à peu de frais, des résultats d'un degré de précision connu et contrôlable. Toutefois, il importe, en appliquant cette théorie, que la sélection de l'échantillon ait lieu strictement d'après certaines règles. Les matières à soumettre au sondage doivent être divisibles en unités de sondage de sorte que chaque partie ait une probabilité connue d'être inclue dans l'échantillon. Ainsi, bien que le charbon ne puisse être soumis au sondage lorsqu'il est entassé, il peut l'être à l'occasion du transport. L'auteur applique la théorie du sondage à la laine, au tabac et au sucre pour la détermination du poids et de la qualité et à l'examen de la condition d'équipements mécanique et électrique. Il expose une méthode pour mesurer séparément l'erreur humaine et l'erreur de sondage.